Oscillatory Subsonic Piecewise Continuous Kernel Function Method

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Theme

TWO main methods currently are employed in the prediction of the aerodynamic forces acting on lifting surfaces at subsonic flow. These methods inlcude the subsonic kernel function approach,[1] which assumes pressure polynomials to describe the pressure field over the wing, and the finite-element approach, such as the doublet lattice method[2] (DLM) or the vortex lattice method (VLM). The kernel function method (KFM), when based on orthogonal polynomials and carefully determined collocation points, shows a rapid convergence of its solution (with a small number of pressure polynomials) provided that the pressure field over the wing is smooth. Pressure discontinuities, such as those arising from control surface rotations, can be treated successfully using the KFM only when the exact shape of the singularity is known.[3] The overlooking of pressure singularities using the KFM leads to a rapid deterioration in the convergence of the solution and to a general loss in the effectiveness of the method. The finite-element methods (FEM) can cope successfully with unknown pressure singularities provided that their location is known. The FEM, however, require a relatively large number of unknowns for convergence, leading at times to a relatively large residual error at the converged values. In the present paper a method is presented, similar to the one described in Ref. 4 in connection with mixed transonic flow, which combines the rapid convergence characteristics of the KFM with the ability of the FEM to treat discontinuities without having to determine their exact form. The method is tested using a two-dimensional airfoil problem (with control surfaces and gaps) with the intention of establishing its merits before embarking on its extension to the three-dimensional flow case.

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Following the DLM, the two-dimensional wing is divided into "boxes," or regions within which the pressure distribution is assumed to be continuous. Therefore, pressure discontinuities are permitted only at the boundaries between these boxes. However, contrary to the DLM, the pressure distribution within each box is not assumed to be constant but is allowed to vary in accordance with a certain polynomial representation. The overall pressure distribution therefore is assumed to be given by a set of piecewise continuous polynomials, with absolutely no continuity requirements between adjoining boxes. Following the KFM, orthogonal polynomials are employed in each of these boxes. Therefore, the leading box polynomials are chosen as orthogonal to the leading-edge (L.E.) singularity, whereas the trailing box polynomials are chosen orthogonal to the trailing-edge (T.E.) singularity. All polynomials relating to intermediate boxes are taken as Legendre polynomials. No allowance is made for any other singularities between the L.E. and T.E. even when control surface rotations are assumed. However, when gaps exist between the main surface of the wing and the control surface, these two surfaces are treated as two separate wings in a manner identical to the one just described.

It seems appropriate, at this stage, to indicate some of the expectations which follow the foregoing approach.

1) The introduction of high-order polynomials in each box is expected to reduce the overall number of boxes so that the total number of unknowns ultimately is reduced.

2) The accuracy of the results is expected to improve, since the employment of orthogonal polynomials in each of the boxes requires the convergence of only the zero- and first-order polynomial coefficients (in each box) for the evaluation of the overall lift, the overall pitching moment, and the control surface hinge moment (in a fashion similar to the Fourier series used by Glauert in the thin airfoil theory[5]).

The polynomial coefficients, for each box, eventually are determined through the use of the downwash collocation method. It can be shown that the overall downwash error is minimized if the collocation points, in each box, are located at the zeros of the downwash polynomial, obtained from a

Table 1 Variation with number of boxes of the percentage error in the aerodynamic coefficients[a]

	2 boxes			4 boxes			6 boxes		
	h	α	β	h	α	β	h	α	β
$Re(C_l)$	0.02	0.01	0.29	0.00	0.06	0.59	0.30	0.03	0.73
$Im(C_l)$	0.00	0.02	0.59	0.04	0.01	0.44	0.06	0.24	1.50
$Re(C_M)$	0.03	0.01	0.19	0.07	0.02	0.20	0.22	0.85	0.44
$Im(C_M)$	...	0.01	0.42	...	0.03	0.63	...	0.10	1.10
$Re(C_H)$	0.01	0.03	0.68	0.04	0.01	2.63	0.07	0.99	4.80
$Im(C_H)$	0.02	0.00	0.19	0.02	0.01	0.48	1.08	0.15	0.48

[a] Total number of pressure polynomials $N=12$. Airfoil with 30% chord T.E. control at $M=0$ and $k=1$.

Table 2 Variation with the number of pressure polynomials per box of the percentage error in the aerodynamic coefficients[a]

	2 polyn. per box			3 polyn. per box			4 polyn. per box		
	h	α	β	h	α	β	h	α	β
$Re(C_L)$	3.11	1.41	1.13	0.14	0.36	0.92	0.02	0.01	0.60
$Im(C_L)$	0.70	2.05	10.30	0.03	0.07	3.21	0.00	0.02	1.56
$Re(C_M)$	0.48	10.10	8.57	0.07	0.45	1.69	0.04	0.03	0.66
$Im(C_M)$	...	1.08	4.66	...	0.09	1.91	...	0.01	1.00
$Re(C_H)$	1.21	8.30	1.54	0.10	0.04	0.75	0.05	0.02	0.95
$Im(C_H)$	6.76	1.11	0.46	0.44	0.25	0.44	0.05	0.02	0.34

[a] Airfoil with 2 boxes and 30% chord T.E. control at $M=0$ and $k=1$.

Table 3 Variation with the number of collocation points in the T.E. box of the percentage error in the aerodynamic coefficients[a]

	5 col. points			6 col. points			7 col. points		
	h	α	β	h	α	β	h	α	β
$Re(C_L)$	0.00	0.03	4.68	0.00	0.03	0.41	0.00	0.03	0.52
$Im(C_L)$	0.01	0.01	8.29	0.01	0.01	0.92	0.01	0.01	1.07
$Re(C_M)$	0.04	0.02	2.45	0.04	0.03	0.34	0.04	0.03	0.37
$Im(C_M)$	...	0.02	6.45	...	0.02	0.62	...	0.02	0.76
$Re(C_H)$	0.00	0.00	13.45	0.00	0.00	0.83	0.01	0.00	1.17
$Im(C_H)$	0.00	0.00	3.43	0.01	0.00	0.26	0.01	0.00	0.34

[a] Airfoil with 30% chord T.E. control, 2 boxes, 5 pressure polynomials per box, and 5 collocation points in the leading box at $M=0$ and $k=1$.

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Table 4 Variation with method of solution of the percentage error in the aerodynamic coefficients[a]

	KFM 10 unknowns			DLM 10 unknowns			Present method 2 bxs., 10 unkns.		
	h	α	β	h	α	β	h	α	β
$Re(C_L)$	0.18	0.05	2.51	122.9	3.27	2.57	0.77	0.06	1.09
$Im(C_L)$	0.05	0.03	3.71	0.38	20.52	10.73	0.04	0.10	2.05
$Re(C_M)$	0.11	0.07	2.93	9.80	8.01	0.15	0.02	0.15	0.79
$Im(C_M)$	0.10	0.08	0.30	0.31	5.14	24.89	0.12	0.02	0.44
$Re(C_H)$	26.41	164.6	266.6	18.40	44.46	47.77	0.26	0.55	1.16
$Im(C_H)$	61.38	44.08	185.9	24.40	20.77	13.03	0.04	0.22	1.41

[a]Airfoil with 20% chord T.E. control at $M=0.8$ and $k=0.88$.

Table 5 Variation with method of solution of the percentage error in the aerodynamic coefficients[a]

Method of solution	VLM	Quasi VLM[7]	Present method
$C_{L\beta}$	6.77	0.67	0.60
$C_{M\beta}$	3.72	0.67	0.87
$C_{H\beta}$	11.88	8.70	2.34

[a]Airfoil with 30% chord T.E. control, 2 boxes with 3 pressure unknowns in L.E. box, 2 pressure unknowns in T.E. box at $M=0$ and $k=0$ (steady flow).

Table 6 Percentage error in the aerodynamic coefficients[a]

Aero. deriv.	$C_{L\alpha}$	$C_{M\alpha}$	$C_{H\alpha}$	$C_{L\beta}$	$C_{M\beta}$	$C_{H\beta}$
% error	0.03	0.75	0.15	0.15	0.14	0.36

[a]Airfoil with 50% chord T.E. control, 1% gap, 2 boxes, 4 pressure polynomials per box at $M=0$ and $k=0$ (steady flow).

pressure polynomial of one order higher than the highest polynomial used in the box. Simple expressions were determined so as to yield good approximations for these zeros of the downwash polynomials (for zero Mach number M and zero reduced frequency k). These expressions were used subsequently for different values of M and k.

It should be mentioned here that a pressure polynomial of order m in the L.E. box yields a downwash polynomial of order m, whereas in both the intermediate and T.E. boxes, it yields a downwash polynomial of order $m+1$. This indicates that these two latter types of boxes will require an additional collocation point in each box for the determination of the downwash polynomial and that a simple least-square procedure should be employed to reduce the number of the resulting equations to the number of unknown pressure coefficients.

In an attempt to test the present method near one of its extreme conditions, results are sought for an airfoil with a T.E. control surface at either $M=0$ and $k=1$, or at $M=0.8$ and $k=0.88$. The results obtained are compared with those given in Ref. 6. For simplicity, all the boxes are allowed the same number of pressure polynomials (with the exception of Table 6 where the comparison requires a different allocation).

These results show that:

1) For a total number of unknowns N (i.e., number of boxes × number of polynomial coefficients per box) kept constant, the best overall accuracy is obtained (Table 1) by taking the smallest number of boxes which is permissible by the present method (equal to 2 for the example shown in Table 1) and increasing the number of pressure polynomials per box.

2) With two boxes representing the airfoil with T.E. control surface (based on item 1 above), a very rapid convergence of the aerodynamic derivatives, with the number of pressure polynomials per box, is obtained (Table 2).

3) A considerable improvement in accuracy is obtained when introducing a single additional collocation point in the T.E. box (Table 3). Further increases in the number of collocation points either in the T.E. box or in the L.E. box yield only negligible changes. These results are in accordance with the preceding discussion regarding the collocation points, and they indicate that collocation points can, in general, be determined in a similar fashion to the one employed in this work.

4) Comparisons are made between the results obtained using the DLM (with 10 boxes), the KFM (with no provisions for the control surface singularity), and the present method (Table 4). Comparison also is made between the Quasi VLM[7] and the present method (Table 5). These results give an indication regarding the effectiveness of the present method. Good results also are obtained (Table 6) when a small gap (1% chord) is allowed to exist between the control surface and the main airfoil surface (comparisons are made in this case with results of Ref. 8).

References

[1]Richardson, J. R., "A Method for Calculating the Lifting Forces on Wings (Unsteady Subsonic and Supersonic Lifting Surface Theory)," Aeronautical Research Council, R&M No. 3157, 1960.

[2]Albano, E. and Rodden, W. P., "A Doublet Lattice Method for Calculating Lift Distribution on Oscillatory Surfaces in Subsonic Flows," *AIAA Journal*, Vol. 7, Feb. 1969, pp. 279-284.

[3]Rowe, W. S., Winther, B. A., and Redman, M. C., "Prediction of Unsteady Aerodynamic Loading Caused by Trailing Edge Control Surface Motions in Subsonic Compressible Flow—Analysis and Results," NASA CR-2003, June 1972.

[4]Cunningham, A. M., "An Oscillatory Kernel Function Method for Lifting Surfaces in Mixed Transonic Flow," AIAA Paper 74-359, April 1974.

[5]Glauert, H., *The Elements of Aerofoil and Airscrew Theory*, Cambridge University Press, 1959.

[6]Jones, W. P., ed., *Manual on Aeroelasticity*, AGARD.

[7]Lan, C. E., "A Quasi-Vortex-Lattice Method in Thin Wing Theory," *Journal of Aircraftf*, Vol. 11, Sept. 1974, pp. 518-527.

[8]White, R. B. and Landahl, M. T., "Effect of Gaps on the Loading Distribution of Planar Lifting Surfaces," *AIAA Journal*, Vol. 6, April 1968, pp. 626-631.